Front propagation in A→2A, A→3A process in $1d$: velocity, diffusion and velocity correlations.

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We study front propagation in the reaction diffusion process $\{A \xrightarrow{\epsilon} 2A, A \xrightarrow{\epsilon_t} 3A\}$ on a one dimensional ($1d$) lattice with hard core interaction between the particles. Using the leading particle picture, velocity of the front in the system is computed using different approximate methods, which is in good agreement with the simulation results. It is observed that in certain ranges of parameters, the front velocity varies as a power law of ϵ_t, which is well captured by our approximate schemes. We also observe that the front dynamics exhibits temporal velocity correlations and these must be taken care of in order to find the exact estimates for the front diffusion coefficient. This correlation changes sign depending upon the sign of $\epsilon_t - D$, where D is the bare diffusion coefficient of A particles. For $\epsilon_t = D$, the leading particle and thus the front moves like an uncorrelated random walker, which is explained through an exact analysis.

I. INTRODUCTION

Front propagation is an important field of study in nonequilibrium systems. We often encounter these propagating fronts separating different phases in physics, chemistry and biology [1]. Here, in this work, we study the dynamics of the front in the reaction-diffusion system $A \rightarrow 2A$, $A \rightarrow 3A$ in one dimensional lattice. At the macroscopic level, the mean field theory yields the following partial differential equation for the coarse grained concentration $\rho(x,t)$,

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2} + 2\epsilon\rho(1-\rho) + \epsilon_t\rho(1-\rho)^2, \qquad (1)$$

where, D is the diffusion coefficient of the particle and ϵ and ϵ_t are the rates of single ($A \rightarrow 2A$) and twin ($A \rightarrow 3A$) offspring production respectively. Equation (1) reduces to the well known Fisher equation [6] when $\epsilon_t = 0$, which models the reaction diffusion equation $A \rightarrow 2A$. The microscopic lattice model for $A \rightarrow 2A$ has been studied extensively [2][3][4][5]. The mean field allows travelling wave solution of the form $\rho(x,t) = \phi(x - vt)$, where the velocity of an initially sharp front between $\rho = 1$ (stable) and $\rho = 0$ (unstable) state approaches an asymptotic velocity $V_0 = 2\sqrt{(2\epsilon + \epsilon_t)D}$.

II. MODEL, FRONT VELOCITY AND DIFFUSION COEFFICIENT

We consider a $1d$ lattice composed of sites $i = 1, 2...L$. We start with the step function like distribution where, the left half is filled with A particles while the right half is empty. Each site i can either be empty or occupied by maximum one particle i.e. hard core exclusion is enforced. We update the system random sequentially where L microscopic moves correspond to one Monte Carlo step (MCS). During each updating we randomly select a site and the particle at the site can undergo one of the following three microscopic moves.

(1) The particle can jump to neighbouring empty site with rate D,
(2) The particle can give birth of one particle at either of the empty neighbouring sites with rate ϵ,
(3) The particle can generate two new particles at both the neighbouring sites provided both are empty with rate ϵ_t.
These processes are shown in the Fig. (1).

As time evolves, these stochastic moves result in the stochastic movement of the front. As has been argued in [5], the front may be identified with the rightmost A particle. In this paper, we are interested in the dynamics of front whose evolution may be described by the following master equation[7].

$$\frac{dP(X,t)}{dt} = (\epsilon + D)P(X-1,t) + \epsilon_t Q_0(X-1,t) +$$
$$DQ_0(X+1,t) - (\epsilon + D)P(X,t) - (\epsilon_t + D)Q_0(X,t) \quad (2)$$

Here, $P(X,t)$ is the probability distribution of finding the front particle at position X at a time t and $Q_0(X,t)$ is the joint probability that the front is at X at time t *and* the site just behind it is empty. In Eq. (2), the first term corresponds to the forward hopping of the front particle from the position $X-1$ to reach X due to birth of a single particle or diffusion. The second term corresponds to twin production at left and right neighbouring sites of site $X-1$ and which results in the front moving from $X-1$ to X. The third term corresponds to the backward hopping of the front particle from position $X+1$ due to diffusion, provided site X is empty. Last two terms account for the possible jumps of the front particle from position X, which leads to front moving either at $X-1$ or $X+1$. The dynamical properties of the front that we want to study are its velocity V and the diffusion coefficient D_f which are defined as:

$$V = \frac{d}{dt} < X(t) > \qquad (3)$$

$$D_f = \frac{1}{2}\frac{d}{dt} < (X(t) - <X(t)>)^2 > \qquad (4)$$

Where, $<X(t)> = \sum_X X P(X,t)$. Now we use the Eq. (2) and normalization $\sum P(X) = 1$ and taking $Q_0(X) = (1 - \rho_{X-1})P(X)$, where ρ_{X-1} is the probability that site $X - 1$ is occupied. Denoting $\rho_{X-1} = \rho_1$, we obtain the following expression for the asymptotic velocity and diffusion coefficient of the front.

$$V = \epsilon + \epsilon_t - \rho_1(\epsilon_t - D) \qquad (5)$$

$$D_f = \frac{1}{2}\{\epsilon + \epsilon_t + 2D - \rho_1(\epsilon_t + D)\} \qquad (6)$$

In order to obtain the velocity and diffusion coefficient we need to know ρ_1, which is the density of site just behind the front. In [2] for $\epsilon_t = 0$, it was shown that front velocity approaches asymptotically the mean field value $V = V_0$ in the limit $\frac{D}{\epsilon} \to \infty$, while $V = \epsilon + D$ in the opposite limit $\frac{D}{\epsilon} \to 0$. But, when we are in between these two extreme limits we need to know ρ_1 and we expect similar features when $\epsilon_t \neq 0$. There is no method to find this value exactly. Here we present some approximate analytic estimates for ρ_1 and hence the front velocity. In subsection A, we use fixed site representation method, where a truncated master equation is written in the frame moving with the front, as discussed in [5]. In subsection B, we apply two particle representation scheme proposed by Kerstein [4] while in C a mixed scheme is proposed which yields better results than either A or B.

A. Fixed site representation

This method has been proposed in [5] for the reaction diffusion process $A \leftrightarrow 2A$. Here, we write a truncated master equation in the frame moving with the front. The simplest set of states is:$\{\circ\bullet, \bullet\bullet\}$, which corresponds to the evolution of occupancy at a site just behind the front particle($l = 1$). Here the rightmost $\bullet$ in each state corresponds to the front particle. These two states make transitions between each other due to the microscopic processes in the system as shown in Fig. (1). Considering all such transitions the evolution of probabilities of these two states are given by:

$$\frac{dP(\circ\bullet)}{dt} = (2D - D\rho_2)P(\bullet\bullet) - \{2D\rho_2 + \epsilon(2 + \rho_2)$$
$$+ \epsilon_t(1 + \rho_2(1 - \rho_3))\}P(\circ\bullet),$$
$$\frac{dP(\bullet\bullet)}{dt} = \{2D\rho_2 + \epsilon(2 + \rho_2) + \epsilon_t(1 + \rho_2(1 - \rho_3))$$
$$\}P(\circ\bullet) - (2D - D\rho_2)P(\bullet\bullet). \qquad (7)$$

Here, ρ_i is the density at the ith site behind the front and we have neglected the spatial density correlation between consecutive pairs of sites beyond the second site behind the front. Now, using Eq. (7) and normalization $P(\circ\bullet) +$
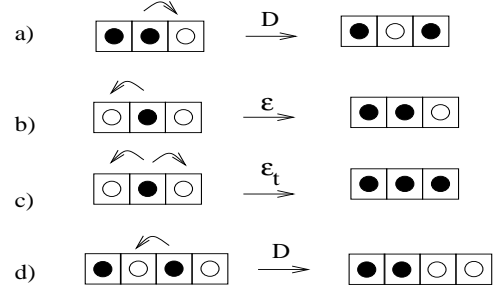

FIG. 1: Microscopic moves, rightmost $\bullet$ represents the front. (a) Diffusion of the front particle to its right site leading to transition from $\bullet\bullet$ to $\circ\bullet$ with rate D. (b) Creation of one particle to the left of the front leads to transition from $\circ\bullet$ to $\bullet\bullet$ with rate ϵ. (c) $\circ\bullet$ changes to $\bullet\bullet$ due to creation of twins at both neighbouring sites of the front with rate ϵ_t. (d) $\circ\bullet \to \bullet\bullet$, if the front takes diffusive move to its left and the second site behind the front is occupied. This occurs with rate $D\rho_2$, where ρ_2 is the probability of occupancy at the second site behind the front.

$P(\bullet\bullet) = 1$, we obtain the following expression for ρ_1:

$$\rho_1 = \frac{2D\rho_2 + \epsilon(2 + \rho_2) + \epsilon_t(1 + \rho_2(1 - \rho_3))}{D\rho_2 + 2D + 2\epsilon + \epsilon\rho_2 + \epsilon_t(1 + \rho_2(1 - \rho_3))} \qquad (8)$$

From Eq. (8), we note that in order to find ρ_1 we need to know ρ_2 and ρ_3. As a crude approximation if we assume that $\rho_2 = \rho_3 = \rho^b = 1$, where ρ^b is the bulk density, we get the following value of ρ_1.

$$\rho_1 \simeq \frac{2D + 3\epsilon + \epsilon_t}{3D + 3\epsilon + \epsilon_t} \qquad (9)$$

Now using this approximation for ρ_1 in Eq. (5), we find the estimate for the velocity which is in reasonable agreement with the simulation, as shown in the Fig. (4) . The estimate for V can be improved by including more sites in the truncated representation. For example, for $l = 2$ we study the evolution of following set of four states: $\{\circ\circ\bullet, \circ\bullet\bullet, \bullet\circ\bullet, \bullet\bullet\bullet\}$ and as expected we get improved results as shown in Fig. (4). Here, we notice that for larger values of ϵ_t the simulation results show nice agreement with that of analytic results. However, as ϵ_t decreases and approaches zero, we see gradual departure of the simulation data from the analytic one. In fact, ρ_i differs from the bulk density significantly with decreasing value ϵ_t as shown in the Fig. (6). That is, the approximation $\rho_i \approx 1$ holds better for larger values of ϵ_t and hence we get better agreement with the simulation results. The estimate for the velocity can be further improved if we include states with larger number of sites. In Fig. (4) we notice two interesting points: firstly, for $D = \epsilon_t$, the theoretical result matches strikingly with the simulation result, secondly, we observe a power law dependence of the velocity on ϵ_t. In fact, the first point, can be shown to be exact by noting that when $D = \epsilon_t$, the front velocity from Eq.(5) is $V = \epsilon + \epsilon_t$, which is independent of
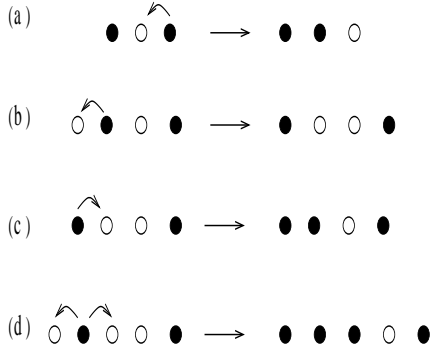
FIG. 2: Transition between two particle states with rightmost ● representing front. (a) Diffusive move of the front particle to its left leading to transition 101→11 with rate D, (b) When the second particle behind the front jumps to the left, provided it is empty, state changes from 101→1001 with rate $D(1-\rho)$, (c) Birth of a single particle by the second particle to its left with rate ϵ leads to transition 1001→101, (d) 1001→101 if the second particle gives birth of two particles, provided the the site left to it is empty, with rate $\epsilon_t(1-\rho)$.

ρ_1.

B. Two particle representation

In the following, we try to find the analytic estimates for ρ_1 using Kerstein's two particles representation[4]. In this representation each state of the system is defined by two rightmost particles and thus we have an infinite set of states: $\{11, 101, 1001, 10001, 100001, ...\}$. Here, the rightmost '1' represents the front particle while the leftmost '1' is the second particle behind it and '0' stands for empty site. Let us denote by P_k the probability of two particle state with k empty sites between the leading particle and next particle behind it. These states form a closed set under transition due to microscopic processes. We have illustrated few transitions in the Fig. (2). Considering all such transitions and denoting the probability of occupancy of site just behind the second particle by ρ, we write the following rate equations for P_k.

$$
\begin{aligned}
\frac{dP_0}{dt} &= (\epsilon + 2D)P_1 + \epsilon_t(1-\rho)P_1 + (2\epsilon + \epsilon_t)(1 - P_0) \\
&\quad -(2D - D\rho)P_0, \\
\frac{dP_k}{dt} &= (2D - D\rho)P_{k-1} + \{\epsilon + 2D + \epsilon_t(1-\rho)\}P_{k+1} \\
&\quad - (4D - D\rho + 3\epsilon + 2\epsilon_t - \epsilon_t\rho)P_k, \quad k \geq 1. \quad (10)
\end{aligned}
$$

In order to solve Eq.(10) we need to specify the dependence of ρ on the parameters$(\epsilon, \epsilon_t, D)$. Following Kerstein [4], we write $\rho = aP_0 - bP_0^2$ and enforcing the condition that $\rho = 1$ when $P_0 = 1$, we write the following expression for ρ.

$$
\rho = (1 + \lambda)P_0 - \lambda P_0^2 \quad (11)
$$

This equation specifies the dependence of ρ on the parameters implicitly through dependence of P_0 on ϵ, ϵ_t, D. Here, λ is a free parameter to be evaluated as follows. Following Kerstein, using the ansatz $P_k = P_0(1 - P_0)^k$ and Eq. (11) in Eq.(10), we get the following quartic equation in P_0.

$$
\begin{aligned}
\epsilon_t\lambda P_0^4 &+ (D\lambda - \epsilon_t - 2\epsilon_t\lambda)P_0^3 + (\epsilon + D + 2\epsilon_t + \epsilon_t\lambda \\
&- D\lambda)P_0^2 + \epsilon P_0 - 2\epsilon - \epsilon_t = 0 \quad (12)
\end{aligned}
$$

In order to find P_0 we need to fix the value of λ. For large D and $\epsilon_t = 0$, it is known that the front particle moves with its mean field velocity[2]. If we assume that this also happens when $\epsilon_t \neq 0$, then equating the mean field front velocity $V_0 = 2\sqrt{(2\epsilon + \epsilon_t)D}$ with that obtained from Eq. (5) i.e. $V \sim DP_0$ when D is very large compared to other parameters, we get $P_0 = 2\sqrt{\frac{2\epsilon+\epsilon_t}{D}}$. Using this value of P_0 in Eq. (12) we find $\lambda = 3/4$ in the limit $D \to \infty$. We solve the quartic equation (12) to get the value of $\rho_1 = P_0$ and hence the front velocity, as shown in the Figs. (4) and (5) and marked as 2P.

C. Mixed representation

Since we are dealing with a multiparticle interacting system it is always desirable to include as many particles as possible while studying the evolution of the system. The simplest extension to the two particle representation is to study the evolution of the following set of states: $\{\circ \bullet \bullet, \bullet \bullet \bullet, \circ \bullet \circ \bullet, \bullet \bullet \circ \bullet ...\}$, where the rightmost $\bullet$ in each state denotes the front particle. Since in this representation, each state is characterized by two or three particles and hence we name it as mixed representation (MR). The rightmost $\bullet$ in each state denotes the front particle. When viewed in the frame moving with the front, each state contains the location of second particle and the occupancy of the site just behind the second particle. We denote these states as $(k, 0)$ or $(k, 1)$, representing the states having k empty sites between the front and the second particle and the site just after the second particle is empty or occupied respectively. For example, by $(0, 0)$ we mean the state $\circ \bullet \bullet$ and $(0, 1)$ for the state $\bullet \bullet \bullet$. These states are making transitions among each other due to the microscopic processes and form a closed set. We have shown some of the transitions in the Fig. (3).

Assuming ρ as the density of site, which is, next nearest neighbour to the second particle, we write the following rate equation for the evolution of probabilities $P(k, 0)$
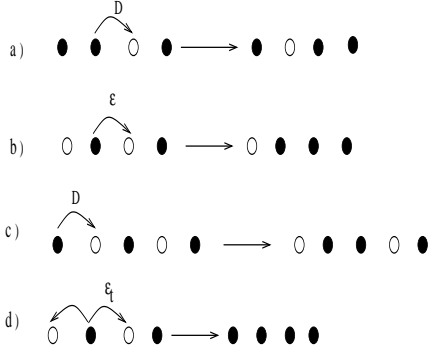
FIG. 3: Transitions between mixed particle states with the rightmost ● representing front. (a) Diffusive move of the particle to the right empty site with rate D.This leads to transition from the state $(1,1)$ to $(0,0)$. (b) Birth of a new particle on the right neighbouring empty site with rate ϵ, which changes the state $(1,0)$ to $(0,1)$. (c) Transition from $(1,0)$ to $(1,1)$ with rate $D\rho$, when the third particle jumps to the right neighbouring empty site. (d) $(1,0)\rightarrow(0,1)$ when the second particle behind the front in $(1,0)$ realization produces twins at the neighbouring empty sites with rate ϵ_t.

FIG. 4: Percentage relative error in V, i.e. $\frac{|V^s - V^a|}{V^s} \times 100$ (V^s and V^a representing simulation and analytic results respectively), is plotted against ϵ_t while keeping D=0.25 and $\epsilon = 0.025$ fixed. Here stars, open squares, open circles and filled circles correspond to the analytic estimates using $l = 2$(two states), $l = 3$ (four states), two particle(2P) and mixed representation (MR) respectively. Simulation profile and analytic profile using MR are essentially coincident and hence we notice almost zero error. Inset: log-log plot for the velocity versus ϵ_t, the straight line shows power law $V \approx \epsilon_t^{0.25}$.

and $P(k,1)$, $k = 0, 1, ..\infty$.

$$
\begin{aligned}
\frac{dP(0,1)}{dt} &= \{D\rho + \epsilon\rho + 2\epsilon + \epsilon_t\rho(1-\rho)\}P(0,0) \\
&+ (D + 2\epsilon + \epsilon_t)P(1,1) + (2\epsilon + 2\epsilon_t)P(1,0) \\
&+ \epsilon_t\{P(2,0) + P(2,1) + P(3,0) + P(3,1) + ...\} \\
&- (2D - D\rho)P(0,1),
\end{aligned}
$$

$$
\begin{aligned}
\frac{dP(0,0)}{dt} &= D(1-\rho)P(0,1) + (D + \epsilon)P(1,1) \\
&+ (2D + \epsilon)P(1,0) + 2\epsilon\{P(2,1) + P(2,0) \\
&+ P(3,0) + P(3,1) + ...\} - \{2D + 2\epsilon + D\rho \\
&+ \epsilon\rho + \epsilon_t\rho(1-\rho)\}P(0,0),
\end{aligned}
$$

$$
\begin{aligned}
\frac{dP(k,1)}{dt} &= DP(k-1,1) + D\rho P(k-1,0) \\
&+ \{D\rho + \epsilon + \epsilon\rho + \epsilon_t\rho(1-\rho)\}P(k,0) \\
&+ (D + \epsilon)P(k+1,1) + (\epsilon + \epsilon_t)P(k+1,0) \\
&- (4D + 3\epsilon - D\rho + \epsilon_t)P(k,1),
\end{aligned}
$$

$$
\begin{aligned}
\frac{dP(k,0)}{dt} &= (D + D(1-\rho))P(k-1,0) + D(1-\rho)P(k,1) \\
&+ DP(k+1,1) + 2DP(k+1,0) - \{4D + 4\epsilon \\
&+ D\rho + \epsilon\rho + 2\epsilon_t + \epsilon_t\rho(1-\rho)\}P(k,0). \quad (13)
\end{aligned}
$$

In order to find P_0, we need to solve the above set of coupled equations. However, one can find the analytic estimate for P_0 by solving rate equations for $P(0,0)$ and $P(0,1)$ and assuming $P(1,1) = \rho P_1, P(1,0) = (1-\rho)P_1$. Using $\sum_{i=0}^{1} P(k,i) = P_k$ and $\sum_{k=0}^{\infty} P_k = 1$, we find steady state expression for $P(0,0)$ and $P(0,1)$ in terms of P_1
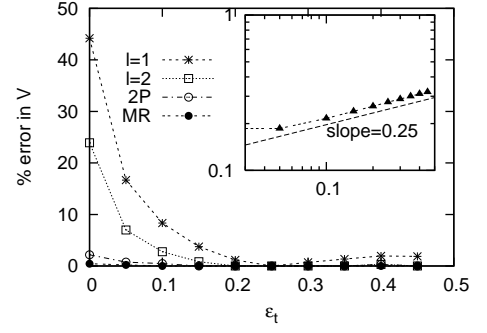
and ρ and then solve the equation:

$$
P(0,0) + P(0,1) = P_0 \quad (14)
$$

Following Kerstein [4], if we use the ansatz $P_1 = P_0(1 - P_0)$, we get the following equation.

$$
\alpha P_0(1 - P_0) + \beta P_0 + \gamma = 0 \quad (15)
$$

Where,

$$
\begin{aligned}
\alpha &= (2\epsilon + \epsilon_t + D\rho - \epsilon_t\rho)\{3D + 2\epsilon + \epsilon\rho + \epsilon_t\rho(1-\rho)\} \\
&+ (2D - D\rho - \epsilon)\{2D + 2\epsilon + \epsilon\rho + \epsilon_t\rho(1-\rho)\}, \\
\beta &= \{2D + 2\epsilon + \epsilon\rho + D\rho + \epsilon_t\rho(1-\rho)\}(2D - D\rho) \\
&- \{D\rho + 2\epsilon + \epsilon\rho + \epsilon_t\rho(1-\rho)\}(D - D\rho), \\
\gamma &= \epsilon_t(1 - P_0)\{3D + 2\epsilon + \epsilon\rho + \epsilon_t\rho(1-\rho)\} \\
&+ 2\epsilon(1 - P_0)\{2D + 2\epsilon + \epsilon\rho + \epsilon_t\rho(1-\rho)\}. \quad (16)
\end{aligned}
$$

The Eq. (15) is in terms of two unknowns ρ and P_0 and hence we need to know ρ in order to find P_0. We specify the dependence of ρ on P_0 similar to what we did in the case of two particle representation. Using the results obtained for $P_0 = \rho_1$ in Eq.(5) we find the estimates for V as shown in the Figs. (4) and (5) marked as MR. We observe good agreement with the simulation results.

III. FRONT DIFFUSION COEFFICIENT

In Fig. (7), we have shown the simulation results of the front diffusion coefficient and compared it with the results obtained by using the mean field and simulation value of ρ_1 in the equation(6). Here, we notice the following interesting features: (1) when $D = \epsilon_t$, the analytical
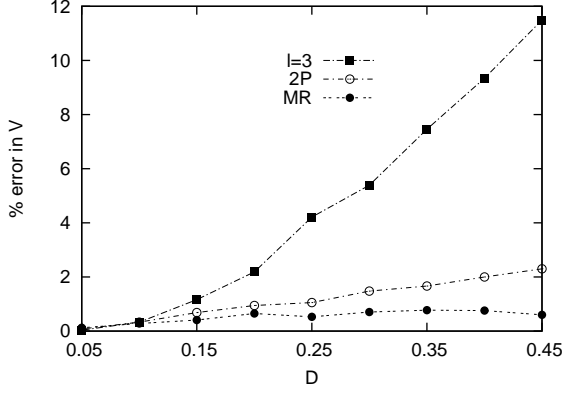
FIG. 5: Percentage relative error in V versus D, keeping $\epsilon = 0.05$ and $\epsilon_t = 0$. The top data(filled squares) corresponds to ref.[5], where in the frame moving with the front ,the evolution of particles at three sites behind the front was studied and assuming the fourth site at the bulk density. The middle data (open circles) corresponds to Kerstein [4] two particle self consistent representation while the bottom data (filled circles) is the result from the mixed representation.
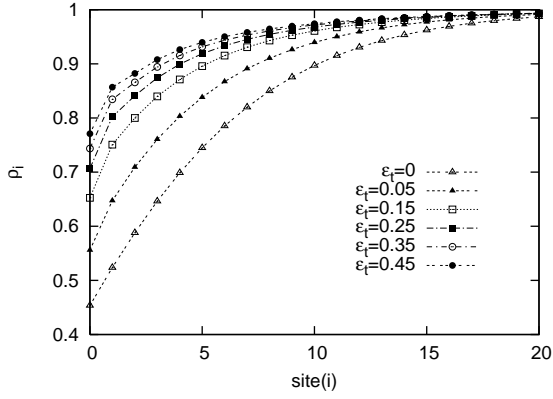

FIG. 7: Comparison of the front diffusion coefficient obtained analytically (from Eq. (6) and using ρ_1 corresponding to $l = 2$) with the simulation results for different values of ϵ_t while keeping $D = 0.25$, ϵ=0.025 fixed. We note that when ϵ_t=D = 0.25, the simulation result matches with the analytic one.

FIG. 6: Density profile behind the front for different values of ϵ_t while keeping $D = 0.25$ and $\epsilon = 0.025$ fixed. Here, we notice that as ϵ_t decreases density profile curve shifts away from the bulk density level.

FIG. 8: Simulation results for velocity correlation of the front with time t, for different values of ϵ_t while keeping $D = 0.25$, ϵ=0.025 fixed. We notice that when $\epsilon_t = 0.25$, this correlation is zero. Inset: log-normal plot for $C(t)$ versus t with $D = 0.05, 0.10, 0.30, 0.40$ from top to bottom.

value D_f^{ana} matches well with the simulation result D_f^{sim}. (2) when $\epsilon_t > D$, the $D_f^{sim} > D_f^{ana}$ (3) when $\epsilon_t < D$, $D_f^{sim} < D_f^{ana}$. The origin of the above discrepancy between the simulation and analytical results can be traced to the master equation (2), where we have neglected the temporal velocity correlations. The expression for the asymptotic front diffusion coefficient with temporal correlations in velocity is given as:

$$D_f = D_0 + \sum_{t=1}^{\infty} C(t) \qquad (17)$$

where, D_0 is the front diffusion coefficient by neglecting correlations as given by Eq. (6) and $C(t)$ is the temporal
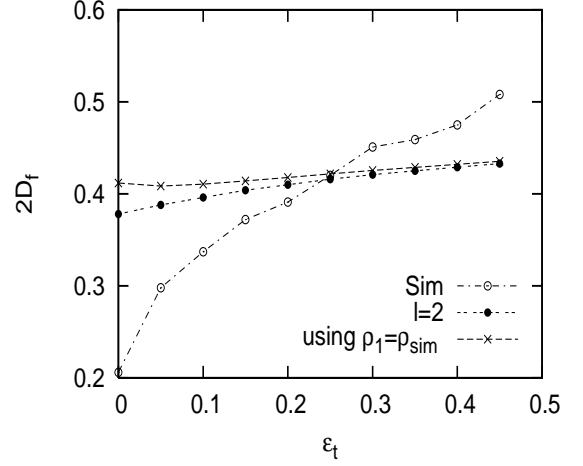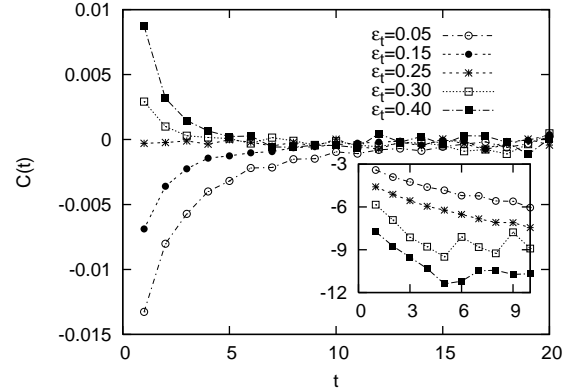
velocity correlation defined through,

$$C(t) = < v(t')v(t' + t) > - < v(t') >< v(t' + t) >, \qquad (18)$$

where, $v(t)$ is the displacement of the front at time t. In Fig. (8), we have plotted the temporal velocity correlation $C(t)$ for different values of ϵ_t. For $\epsilon_t > D$, we observe positive correlation while for $\epsilon_t < D$, it is negative and for $\epsilon_t = D$, $C(t)$ seems to vanish for all t. Thus, $\epsilon_t = D$ is a special case, where the front particle moves like an uncorrelated random walker.

In the following, we explicitly show that for the special case $\epsilon_t = D$, two consecutive steps of the leading

particle are uncorrelated, i.e., $C(1) = 0$ in the steady state. Since at most two sites behind the front can be affected in two consecutive steps, we consider 4 states corresponding to $l = 2$, namely, $\{001, 011, 101, 111\}$ with the rightmost $'1'$ representing the front. In order to find $C(1) = <v(t)v(t+1)> - <v(t)><v(t+1)>$, we write $<v(t)v(t+1)> = R_{++} - R_{+-} - R_{-+} + R_{--}$, where R_{ij} denotes the 'flux' $R_{ij}^{001} + R_{ij}^{011} + R_{ij}^{101} + R_{ij}^{111}$ for taking two consecutive steps as $i = +/-$ and $j = +/-$. Here, for example, R_{--}^{001} is the flux of two consecutive negative steps starting from the state 001. The only way it can occur is if the front particle takes two diffusive moves to the left and thus, $R_{--}^{001} = D^2 P_{001}$, where P_{001} is the steady state weight of the configuration 001. Considering all such two successive moves in the each state, we write the following expression for R_{++}, R_{+-}, R_{-+} and R_{--},

$$
\begin{aligned}
R_{++} &= D^2 + 2\epsilon D + \epsilon_t D + \epsilon^2 + (\epsilon_t D + \epsilon_t \epsilon)\{P_{001} + P_{101}\}, \\
R_{+-} &= D^2, \\
R_{-+} &= (D^2 + D\epsilon + D\epsilon_t)P_{001} + (D^2 + D\epsilon)P_{101}, \\
R_{--} &= D^2 P_{001}.
\end{aligned}
\tag{19}
$$

Now using Eqs.(19), we have

$$
\begin{aligned}
<v(t)v(t+1)> &= \epsilon^2 + D\epsilon_t + 2\epsilon D + \{\epsilon_t D + \epsilon_t \epsilon \\
-D^2 - D\epsilon\}P_{101} &+ \{\epsilon_t \epsilon - D\epsilon\}P_{001},
\end{aligned}
\tag{20}
$$

and similarly,

$$
<v(t)> = <v(t+1)> = \epsilon + \epsilon_t - \{P_{011} + P_{111}\}(\epsilon_t - D).
\tag{21}
$$

Using Eqs.(20) and (21) we can find the value of $<v(t)v(t+1)>$ or $<v(t)>$ by finding the probabilities of different states, which is harder to compute exactly. However, when $\epsilon_t = D$, we find that $<v(t)v(t+1)> - <v(t)><v(t+1)>$ is independent of all the probabilities and is equal to zero, i. e., two successive steps are uncorrelated as observed in the simulation result (Fig.8). We also note that for this special case the above analy-sis does not involve any approximation i.e. it is exact. We also notice that when $\epsilon_t \neq D$, $<v(t)v(t+1)> \neq <v(t)><v(t+1)>$, i.e., the front motion is correlated. Preliminary fits suggest that the the temporal velocity correlation has the form, $C(t) \sim t^\alpha e^{-\beta t}$.

IV. CONCLUSION

In the present paper, we studied the reaction diffusion system $A \to 2A, A \to 3A$ in one dimension. Treating the rightmost occupied site as a front we compute the front velocity analytically using different approximate methods. In fixed site representation one can systematically improve upon the estimate by studying the evolution of particles at larger number of sites behind the front. The results from two particle representation and mixed representation show excellent agreement with the simulation results. We also observed that the velocity depends on ϵ_t as a power law. As far as the computation of front diffusion coefficient is concerned, we notice that one needs to take into account the temporal velocity correlation. In fact the observed temporal correlations in the front dynamics changes sign with sign of $\epsilon_t - D$. For $\epsilon_t > D$ or $\epsilon_t < D$, front moves like a positively or negatively correlated random walk and for $\epsilon_t = D$, the temporal correlations in steps vanish and the front particle moves like a simple uncorrelated random walker. An interesting generalization of the process would be to include the annihilation of particles as well i.e., $2A \to A$ with rate W. The case $\epsilon_t = 0$ and $W = D$ is exactly solvable [8] and both temporal and spatial correlations vanish. For non-zero D, ϵ_t and W, simulations show that temporal correlations vanish on the plane $D = \epsilon_t + W$, although spatial correlations do not. Computation of the previous section can be extended to this case to show that consecutive steps are temporally uncorrelated. A general proof to show that $C(t)$ vanishes for all t remains an interesting open problem.

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